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FACULTY OF MATHEMATICS
AND COMPUTER SCIENCE
University of Lodz


Program \& Abstracts

# The 10th Visegrad Conference on Dynamical Systems June 29 - July 1, 2023, Łódź, Poland 

## Main speakers

- Henk Bruin University of Vienna, Austria
- Dikran Dikranjan University of Udine, Italy
- Dominik Kwietniak Jagiellonian University, Poland
- Piotr Oprocha AGH University of Science and Technology, Poland
- Klaus Schmidt University of Vienna, Austria
- Sonja Štimac University of Zagreb, Croatia
- Hiroki Sumi Kyoto University, Japan
- Anna Zdunik University of Warsaw, Poland


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## List of TALKS

## INVITED TALKS

| Henk Bruin | Dynamical and ergodic properties of rotated odometers <br> Dikran Dikranjan |
| :--- | :--- |
| Entropy of amenable monoid actions |  |
| Dominik Kwietniak | An anti-classification theorem for the topological conjugacy problem <br> of Cantor minimal systems |
| Piotr Oprocha | On planar attractors and inverse limits |
| Klaus Schmidt | Generators and symbolic representations of algebraic actions <br> The pruning front conjecture and classification of the Hénon maps <br> in the presence of strange attractors |
| Sonja Štimac | Random dynamical systems of polynomial automorphisms on $\mathbb{C}^{2}$ <br> Hiroki Sumi <br> Anna Zdunik |

## Contributed talks

Sergey Bezuglyi IFS measures on Bratteli diagrams
Zoltán Buczolich
Magdalena Foryś-
Krawiec
Domagoj Jelić
Agnieszka Marczuk
Michaela Mihoková
Mark Pollicott
Silvia Radinger
Lenka Rucká
Daniel Sell
L'ubomír Snoha
Paweł Walczak

Measures, annuli and dimensions
Homeo-product-minimality of the pseudo-circle
On recurrence and entropy in hyperspace of continua in dimension one
Bowen's equation for a dynamical solenoid
Free interval, retracts and minimality
Complex dimensions for dynamically defined Cantor sets
Interval translation maps with weakly mixing attractors
Minimality and distributional chaos in triangular maps
On invariant measures of $\mathscr{B}$-free subshifts
Rigidity and flexibility of polynomial entropy
Some consequences of Hahn-Banach Theorem

## Program

## Thursday, June 29th

8:30-9:50 Registration
9:50-10:00 Opening
10:00-10:50 Hiroki Sumi Random dynamical systems of polynomial automorphisms on $\mathbb{C}^{2}$
11:00-11:30 Coffee break
11:30-11:55 Silvia Radinger Interval translation maps with weakly mixing attractors
12:00-12:25 Sergey Bezuglyi IFS measures on Bratteli diagrams
12:30-12:55 Lenka Rucká Minimality and distributional chaos in triangular maps
13:00-15:00 Lunch at Arche Vita Restaurant, Matejki 11
15:00-15:50 Dominik Kwietniak An anti-classification theorem for the topological conjugacy problem of Cantor minimal systems

16:00-16:30 Coffee break
16:30-16:55 Magdalena Foryś-Krawiec Homeo-product-minimality of the pseudo-circle
17:00-17:25 Michaela Mihoková Free interval, retracts and minimality

## Friday, June 30th

9:00- 9:50 Henk Bruin Dynamical and ergodic properties of rotated odometers
10:00-10:50 Klaus Schmidt Generators and symbolic representations of algebraic actions
11:00-11:30 Coffee break
11:30-11:55 Paweł Walczak Some consequences of Hahn-Banach Theorem
12:00-12:25 Daniel Sell On invariant measures of $\mathscr{B}$-free subshifts
12:30-12:55 Zoltán Buczolich Measures, annuli and dimensions
13:00-15:00 Lunch at Arche Vita Restaurant, Matejki 11
15:00-15:50 Sonja Štimac The pruning front conjecture and classification of the Hénon maps in the presence of strange attractors

19:00-22:00 Banquet at Arche Vita Restaurant, Matejki 11

## Saturday, July 1st

9:00- 9:50 Anna Zdunik Random dynamics of polynomial and entire maps
10:00-10:50 Dikran Dikranjan Entropy of amenable monoid actions
11:00-11:30 Coffee break
11:30-11:55 L'ubomír Snoha Rigidity and flexibility of polynomial entropy
12:00-12:25 Domagoj Jelić On recurrence and entropy in hyperspace of continua in dimension one

12:30-12:55 Agnieszka Marczuk Bowen's equation for a dynamical solenoid
13:00-15:00 Lunch at Arche Vita Restaurant, Matejki 11
15:00-15:50 Piotr Oprocha On planar attractors and inverse limits
16:00-16:30 Coffee break
16:30-16:55 Mark Pollicott Complex dimensions for dynamically defined Cantor sets
17:00-17:10 Closing

## Abstracts

# IFS measures on Bratteli diagrams 

Sergey Bezuglyi<br>University of Iowa, USA

In my talk, I will consider self-similar measures (called also iterated function system measures) on the path space of generalized Bratteli diagrams. By a generalized Bratteli diagram we mean Bratteli diagrams whose levels are represented either by a countable set or by a standard Borel space. We study new classes of iterated function systems related to stationary generalized Bratteli diagrams. For path space systems, in our main result, we give a necessary and sufficient condition for the existence of such generalized IFS measures. For the corresponding iterated function systems, we further identify the measures which are also shift-invariant.

The talk is based on a joint paper with Palle Jorgensen, arXiv:2210.14059.

# Dynamical and ergodic properties of rotated odometers 

Henk Bruin<br>University of Vienna, Austria

In joint works with Olga Lukina, I studied a class of infinitely interval exchange transformations consisting of a composition of the Kakutani-Von Neuman map (odometer) and a (rational) interval exchange transformation. These also represent translation flows on certain surfaces of infinite genus.

In this talk I want to survey some of their topological and ergodic features of these systems.

Measures, annuli and dimensions<br>Zoltán Buczolich<br>ELTE Eötvös Loránd University, Hungary

Given a Radon probability measure $\mu$ supported in $\mathbb{R}^{d}$, we are interested in those points $x$ around which the measure is concentrated infinitely many times on thin annuli centered at $x$. Depending on the lower and upper dimension of $\mu$, the metric used in the space and the thinness of the annuli, we obtain results and examples when such points are of $\mu$-measure 0 or of $\mu$-measure 1 .

The measure concentration we study is related to "bad points" for the Poincaré recurrence theorem and to the first return times to shrinking balls under iteration generated by a weakly Markov dynamical system.

The study of thin annuli and spherical averages is also important in many dimensionrelated problems, including Kakeya-type problems and Falconer's distance set conjecture.

This talk is based on a joint paper with Stéphane Seuret.

# Entropy of amenable monoid actions 

Dikran Dikranjan (1), Anna Giordano Bruno (1), Simone Virili (2)
(1) Udine University, Italy, (2) Autonomous University of Barcelona, Spain

For a right action $K \stackrel{\rho}{\curvearrowleft} S$ of a cancellative right amenable monoid $S$ on a compact Hausdorff space $K$, we build its Ore colocalization $K^{*} \stackrel{\rho^{*}}{\curvearrowleft} G$, where $K^{*}$ is a compact space and $G$ is the group of left fractions of $S$. This construction preserves the topological entropy (i.e., $h_{\text {top }}\left(\rho^{*}\right)=$ $\left.h_{\text {top }}(\rho)\right)$ and linearity of the action.

Similarly, for a left linear action $S \stackrel{\lambda}{\curvearrowright} X$ on a discrete Abelian group $X$, we construct its Ore localization $G \stackrel{\lambda^{*}}{\curvearrowright} X^{*}$, which is linear and preserves the algebraic entropy $h_{\text {alg }}$ (i.e., $h_{\text {alg }}\left(\lambda^{*}\right)=$ $h_{\text {alg }}(\lambda)$ ). Moreover, if $K \stackrel{\rho}{\curvearrowleft} S$ a right linear action with $K$ a compact Abelian group and $S \stackrel{\rho^{\wedge}}{\curvearrowright} X$ is the dual left action on the discrete Pontryagin dual $X:=K^{\wedge}$, then the Ore localization of $\rho^{\wedge}$ is conjugated to dual of the Ore colocalization $K^{*} \stackrel{\rho}{\curvearrowleft} G$. Using this fact, we prove the useful equality $h_{\text {top }}(\rho)=h_{\text {alg }}\left(\rho^{\wedge}\right)$, known also as Bridge Theorem.

We obtain an Addition Theorem for $h_{\text {top }}$ (i.e., for a linear action $K \stackrel{\rho}{\curvearrowleft} S$ on a compact group $K$, a $\rho$-invariant closed subgroup $H$ of $K$ and the left cosets space $K / H, h_{\text {top }}(\rho)=h_{\text {top }}\left(\rho_{H}\right)+$ $\left.h_{\text {top }}\left(\rho_{K / H}\right)\right)$, as well as a similar Addition Theorem for $h_{\text {alg }}$.

## Homeo-product-minimality of the pseudo-circle

Magdalena Forýs-Krawiec<br>AGH University of Krakow, Poland

A compact space $Y$ is called homeo-product-minimal if, given any minimal system ( $X, T$ ), it admits a homeomorphism $S: Y \rightarrow Y$ such that the product system $(X \times Y, T \times S)$ is minimal. The idea of homeo-product-minimality is motivated by the fact, that there exist minimal spaces, whose Cartesian powers do not admit minimal homeomorphisms.

In the talk we present the following result, based on the modification of Handel's construction of the pseudo-circle:

Theorem. The pseudo-circle is homeo-product-minimal.
The results presented during the talk are obtained as a joint work with Jan Boroński and Piotr Oprocha.

# On recurrence and entropy in hyperspace of continua in dimension one <br> Domagoj Jelić <br> University of Split 

Whenever we are given a selfmap $f$ of a compact metric space $X$, we can associate with it the induced mappings $\bar{f}$ and $\tilde{f}$ on the hyperspace $2^{X}$ of compact subsets of $X$ and the hyperspace $C(X)$ of continua in $X$, respectively, both defined in a natural way.

In this talk we discuss and provide the affirmative answer to the following question:
If $G$ is a topological graph, and $f$ is continuous map, does the induced map $\tilde{f}$ acting on the hyperspace $C(G)$ carry the same entropy as $f$ ?

It is well known that this does not hold on the larger hyperspace of all compact subsets. Also negative examples were given for the hyperspace $C(X)$ on some continua $X$, including dendrites.

Moreover, full characterization of the set of recurrent points of $(C(G), \tilde{f})$ is provided. These results extend previous positive results obtained first for much simpler case of compact interval by completely different tools.

The talk is based on a joint work with Piotr Oprocha.

# An anti-classification theorem for the topological conjugacy problem of Cantor minimal systems 

Dominik Kwietniak

Jagiellonian University, Poland

The isomorphism problem in dynamics dates back to a question of von Neumann from 1932: Is it possible to classify (in some reasonable sense) the ergodic measure-preserving diffeomorphisms of a compact manifold up to isomorphism? We would like to study a similar problem: let $C$ be the Cantor set and let $\operatorname{Min}(C)$ stand for the space of all minimal homeomorphisms of the Cantor set. Recall that a Cantor set homeomorphism $f$ is in $\operatorname{Min}(C)$ if every orbit of $f$ is dense in $C$. We say that $f$ and $g$ in $\operatorname{Min}(C)$ are topologically conjugate if there exists a Cantor set homeomorphism $h$ such that $f \circ h=\circ g$. We prove an anti-classification result showing that even for very liberal interpretations of what a "reasonable" classification scheme might be, a classification of minimal Cantor set homeomorphism up to topological conjugacy is impossible. We see it as a consequence of the following: we prove that the topological conjugacy relation of Cantor minimal systems TopConj treated as a subset of $\operatorname{Min}(C) \times \operatorname{Min}(C)$ is complete analytic. In particular, TopConj is a non-Borel subset of $\operatorname{Min}(C) \times \operatorname{Min}(C)$. Roughly speaking, it means that it is impossible to tell if two minimal Cantor set homeomorphisms are topologically conjugate using only a countable amount of information and computation.

Our result is proved by applying a Foreman-Rudolph-Weiss-type construction used for an anti-classification theorem for ergodic automorphisms of the Lebesgue space. We find a continuous map $F$ from the space of all trees over non-negative integers with arbitrarily long branches into the class of minimal homeomorphisms of the Cantor set. Furthermore, $F$ is a reduction, which means that a tree $T$ is ill-founded if and only if $F(T)$ is topologically
conjugate to its inverse. Since the set of ill-founded trees is a well-known example of a complete analytic set, we see that it is essentially impossible to classify which minimal Cantor set homeomorphisms are topologically conjugate to their inverses.

This is joint work with Konrad Deka, Felipe García-Ramos, Kosma Kasprzak, Philipp Kunde (all from the Jagiellonian University in Kraków).

## Bowen's equation for a dynamical solenoid

Agnieszka Marczuk<br>University of Lodz, Poland

The first connection between topological pressure of a classical dynamical system and Haussdorf dimension of its repeller was proved by Bowen who showed that for some transformation of the Riemann sphere and its repeller being the quasi-circle J, the Haussdorf dimension $t=\operatorname{dim}_{H}(J)$ is the unique root of a preesure function. There are several generalization of this result, respectively by Ruell, Rugh and Climenhaga. Under mild assumption, we prove that similar result holds for a dynamical solenoid.

The talk is based on the joint work with A.Biś and W. Kozłowski.

# Free interval, retracts and minimality 

Michaela Mihoková

Matej Bel University, Slovakia
A free interval is an open set homeomorphic to the real interval $(0,1)$. An example of a nontrivial space having such an interval is the Warsaw circle. Dynamics on spaces with a free interval was studied, e.g., in $[3,4,5,6]$. For dynamics on almost meshed continua (i.e., continua having dense union of free intervals), see [1, 2].

In this talk we will speak about minimal sets on continua having a dense free interval. In a particular case we will obtain a full characterization of the topological structure of minimal sets. One of the main ingredients of the proof is the notion of retract and the result of Kuratowski and Dugundji stating that every locally connected subcontinuum with a onedimensional complement is a retract of the space. This talk will be based on [7].

## References

[1] G. Acosta, R. Hernández-Gutiérrez, I. Naghmouchi, P. Oprocha, Periodic points and transitivity on dendrites, Ergodic Theory Dynam. Systems 37 (2017), no. 7, 2017-2033.
[2] V. Córdova-Salazar, D. Herrera-Carrasco, F. Macías-Romero, Almost meshed locally connected continua have unique third symmetric product, Topology Appl. 268 (2019), 11 pp.
[3] M. Dirbák, R. Hric, P. Maličký, L’. Snoha, V. Špitalský, Minimality for actions of abelian semigroups on compact spaces with a free interval, Ergodic Theory Dynam. Systems 39 (2019), no. 11, 2968-2982.
[4] M. Dirbák, L'. Snoha, V. Špitalský, Minimality, transitivity, mixing and topological entropy on spaces with a free interval, Ergodic Theory Dynam. Systems 33 (2013), no. 6, 1786-1812.
[5] G. Harańczyk, D. Kwietniak, P. Oprocha, A note on transitivity, sensitivity and chaos for graph maps, J. Difference Equ. Appl. 17 (2011), no. 10, 1549-1553.
[6] K. Kawamura, A direct proof that each Peano continuum with a free arc admits no expansive homeomorphisms, Tsukuba J. Math. 12 (1988), no. 2, 521-524.
[7] M. Mihoková, Minimal sets on continua with a dense free interval, J. Math. Anal. Appl. 517 (2023), no. 1, 17 pp.

# On planar attractors and inverse limits 

Piotr Oprocha
AGH University, Poland
A very useful technique called BBM (Brown-Barge-Martin), incorporates inverse limits and natural extensions of the underlying bonding maps to embed attractors in manifolds. The original idea goes back to the paper of Barge and Martin, where the authors constructed strange attractors from a wide class of inverse limits. One of the crucial steps for this technique to work is the usage of Brown's approximation theorem. Recently, this technique was extended to produce a parameterized family of strange attractors. In this talk we will present a few possible applications of BBM technique in construction of concrete examples.

This is joint work with Jernej Činč.
P.O. was partially supported by National Science Centre, Poland (NCN), grant no. 2019/35/B/ST1/02239.

# Complex dimensions for dynamically defined Cantor sets 

Mark Pollicott<br>University of Warwick, United Kingdom

Lapidus associated to certain Cantor sets in the unit interval the notion of "complex dimensions". These are complex numbers which are defined using (a Dirichlet series involving) the lengths of the intervals in the complement of the Cantor set. We will describe how one might study these in the context of appropriate dynamically defined sets (i.e., attractors of iterated function schemes). In particular, as an illustration we will apply this to Cantor sets of points whose continued fraction expansion consists of coefficients chosen from a finite set.

# Interval translation maps with weakly mixing attractors <br> Silvia Radinger <br> University of Vienna, Austria 

In 2003 H . Bruin and S. Troubetzkoy studied a renormalization map for a two-parameter family of interval translation maps. For a non-typical subset of the parameter space the
interval translation map has a Cantor attractor. The renormalization $G$, a procedure similar to the Rauzy induction, acts as dynamics on the parameter space and can be used to decide the attractor and in the case of a Cantor attractor, whether the interval translation map is uniquely ergodic.

In this talk we further study these systems, focusing on weak mixing. We look the symbolic representation of the interval translation map to define a S-adic subshift and use results about the eigenvalues of Bratteli-Vershik systems to Further we characterize the subset of linearly recurrent interval translation maps and their eigenvalues.

This is a joint work with Henk Bruin.

# Minimality and distributional chaos in triangular maps 

Lenka Rucká<br>Silesian University in Opava

It was proved by Paganoni and Smítal in [1], that if a triangular map $F$ of the square is nondecreasing on fibers and holds the property that any $\omega$-limit set contains a unique minimal set, then $F$ is not DC1-chaotic. In this talk we will prove the same result for wider class of maps - any triangular map of the square.

In [2], Balibrea and Smítal tried to construct a DC1 triangular homeomorphism monotone on fibers (not only non-decreasing), such that any $\omega$-limit set contains a unique minimal set. We will show that their construction leads to a non-continuous map and therefore it cannot be used as a counterexample to our result.

Joint work with Francisco Balibrea.

## References

[1] L. Paganoni, J. Smítal; Strange distributionally chaotic triangular maps, Chaos Solitons Fractals 26 (2005), no. 2, 581-589.
[2] F. Balibrea, J. Smítal; Strong distributional chaos and minimal sets, Topology appl. 156 (2009), 1673-1678.

## Generators and symbolic representations of algebraic actions

Klaus Schmidt<br>University of Vienna, Austria

Expansive algebraic actions of infinite groups have very natural symbolic representations (e.g. Markov partitions of hyperbolic toral automorphisms). Here we construct symbolic representations of intrinsically ergodic, but not necessarily expansive, algebraic actions of infinite amenable groups and use these representations to find explicit generating partitions (up to null-sets) for such actions.

This is joint work with Hanfeng Li.

# On invariant measures of $\mathscr{B}$-free subshifts <br> Daniel Sell <br> Nicolaus Copernicus University, Poland 

For a set $\mathscr{B} \subseteq \mathbb{N}$, the characteristic function of $\mathscr{B}$-free numbers defines via its orbit closure the so-called $\mathscr{B}$-free subshift $X_{\eta} \subseteq\{0,1\}^{\mathrm{Z}}$. Its (unique) minimal component is a Toeplitz subshift generated by a $\mathscr{B}$-free Toeplitz sequence $\eta^{*}$. For taut $\mathscr{B}$, the elements of $X_{\eta}$ are precisely those sequences that lie between $\eta^{*}$ and $\eta$. I will present results from joint work with Aurelia Dymek and Joanna Kułaga-Przymus where we use this characterisation. Under the assumption that $\eta^{*}$ is a regular Toeplitz sequence, we obtain statements about elements, entropy and measures of $X_{\eta}$, analogous to descriptions that were previously only known for the hereditary closure of $X_{\eta}$.

# Rigidity and flexibility of polynomial entropy 

L'ubomír Snoha<br>Matej Bel University, Slovakia<br>(joint work with Samuel Roth and Zuzana Roth)

We consider dynamical systems $(X, f)$ given by a continuous selfmap $f$ of a compact metric space $X$. Flexibility means that for a given class of dynamical systems a considered dynamical invariant can take arbitrary values, subject only to natural restrictions; flexibility as a program in dynamics was recently formulated by A. Katok. Rigidity in this talk means that a considered dynamical invariant can take only very restricted values for a given class of systems.

Systems with zero topological entropy may still exhibit complicated behaviors. Polynomial entropy, as one of the so called slow entropies, can be used to measure the complexity of dynamical systems in the zero entropy regime. By definition, the polynomial entropy of ( $X, f$ ) is

$$
h_{\mathrm{pol}}(f)=\lim _{\varepsilon \rightarrow 0} \limsup _{n \rightarrow \infty} \frac{\log \operatorname{sep}(n, \varepsilon, f)}{\log n} .
$$

where $\operatorname{sep}(n, \varepsilon, f)$ is the maximal cardinality of subsets of $X$ which are $(n, \varepsilon)$-separated for $f$ (recall that in the definition of the topological entropy there is $n$ in the denominator, instead of $\log n$ ). Thus, polynomial entropy measures the polynomial growth rate of distinguishable orbit segments. It is infinite whenever topological entropy is positive, and so it is useful only for systems with zero topological entropy.

In [1] we show that polynomial entropy has both rigidity and flexibility aspects. In general it is flexible - it may take any value in $[0, \infty]$, even for homeomorphisms on continua (also for continuous dendrite maps it may take many non-integer values). However, for continuous selfmaps of the interval the polynomial entropy is rigid, taking only nonnegative integer values, including infinity. To prove this, we introduce the notion of a one-way horseshoe and show that the polynomial entropy of an interval map equals the supremum of the lengths
of one-way horseshoes of the map and its iterates (an analogue of Misiurewicz's theorem on topological entropy and standard 'two-way' horseshoes). This already implies the rigidity result. As another application of the developed theory we compute the polynomial entropy of all maps in the logistic family.

## References

[1] S. Roth, Z. Roth, L'. Snoha, Rigidity and flexibility of polynomial entropy, 33 pages, submitted.

## The pruning front conjecture and classification of the Hénon maps in the presence of strange attractors

Sonya Štimac
University of Zagreb, Croatia
I will talk about recent results on topological dynamics of the Hénon maps obtained in joint work with Jan Boroński. For a parameter set generalizing the Benedicks-Carleson parameters (the Wang-Young parameter set) we obtain the following: The pruning front conjecture (due to Cvitanović, Gunaratne, and Procacci); A kneading theory (realizing a conjecture by Benedicks and Carleson); A classification: two Hénon maps are conjugate on their strange attractors if and only if their sets of kneading sequences coincide, if and only if their folding patterns coincide. The classification result relies on a further development of the authors' recent inverse limit description of the Hénon attractors in terms of densely branching trees.

## Random dynamical systems of polynomial automorphisms on $\mathbb{C}^{2}$

Hiroki Sumi<br>Kyoto University, Japan

We consider i.i.d. random dynamical systems of polynomial automorphisms on $\mathbb{C}^{2}$. In particular, we consider i.i.d random dynamical systems of complex generalized Hénon maps and their conjugated maps on $\mathbb{C}^{2}$. We show that for a generic such system, we have the following.
(1) There exists only finitely many minimal sets $L_{1}, \ldots, L_{m}$ in $\mathbb{C}^{2}$, and each $L_{j}$ is attracting.
(2) For each initial value $z$ in $\mathbb{C}^{2}$ and for almost every sequence of maps $\gamma=\left(\gamma_{1}, \gamma_{2}, \ldots\right)$, the orbit $\left\{\gamma_{n} \cdots \gamma_{1}(z)\right\}_{n=1}^{\infty}$ tends to a point in the line at infinity or tends to one of $L_{1}, \ldots, L_{m}$.
(3) For each initial value $z$ in $\mathbb{C}^{2}$ and for almost every sequence of maps $\gamma=\left(\gamma_{1}, \gamma_{2}, \ldots\right)$, the Lyapunov exponent of $\gamma$ at $z$ with respect to the Fubini-Study metric on $\mathbb{P}^{2}$ is negative.
Note that the above phenomenon cannot hold for deterministic dynamical systems of iterations of single complex generalized Hénon maps. Thus we see a randomness-induced phenomenon (a phenomenon of random dynamical systems which cannot hold for deterministic dynamical systems of iterations of single complex generalized Hénon maps).

# Some consequences of Hahn-Banach Theorem 

Paweł Walczak<br>University of Lodz, Poland

We shall show how to use Hahn-Banach Theorem in proving existence of particular measures (invariant, harmonic, etc.) for classical (single transformations and flows) and generalized (groups, semigroups, pseudogroups, foliations) dynamical systems.

## Random dynamics of polynomial and entire maps

Anna Zdunik
Warsaw University, Poland
The study of random dynamics of holomorphic maps in the Riemann sphere was inspired by the seminal paper of E . Fornaess and N. Sibony.

I will present some results about random dynamics of polynomials and entire maps. For example, I will consider random (or: non- autonomous) iteration of maps in quadratic family $Q_{c}(z)=z^{2}+c$, and random iteration in the exponential family $E(z)=\exp (z)$. In particular, the following questions will be addressed: connectedness of the Julia set, Hausdorff dimension of the Julia set and dimension of the radial Julia set and its dependence on the "range of randomness", dimension of the harmonic measure on the Julia set of random iteration of polynomials.

## List of participants

1. Sejal Babel
2. Sergey Bezuglyi
3. Andrzej Biś
4. Henk Bruin
5. Zoltśn Buczolich
6. Udayan Darji
7. Dikran Dikranyan,
8. Tomasz Downarowicz
9. Magdalena Foryś-Krawiec
10. Bartosz Frej
11. Grażyna Horbaczewska
12. Roman Hric
13. Domagoj Jelić
14. Olena Karpel
15. Dominik Kwietniak
16. Agnieszka Marczuk
17. Eugen Mihailescu
18. Michaela Mihoková
19. Radu Munteanu
20. Anima Nagar
21. Piotr Oprocha
22. Mark Pollicott
23. Habibeh Pourmand
24. Michał Prusik
25. Silvia Radinger
26. Lenka Rucka
27. Klaus Schmidt
28. Daniel Sell
29. Jaqueline Siqueira
30. L’ubomír Snoha
31. Sonja Štimac
32. Hiroki Sumi
33. Paulo Varandas
34. Paweł Walczak
35. Benjamin Weiss
36. Anna Zdunik

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